3.4 Radical and Rational Exponent Equations MATH 161 THOMPSON

1) $(-9)^2 = 81$ $-9^2 = -81$ **second one, the negative isn't squared*

2) Evaluate $\sqrt{\mathbf{100}}$ and $\sqrt{(-\mathbf{10})^2}$ = ***both equal 10***

3) $\sqrt[n]{a^m}$ n is called the index.

4) True or False $\sqrt[4]{(-2)^4} = -2$ FALSE squaring a negative will always positive

5) $\sqrt{81} + \sqrt{16} = 9 + 4 = 13$

6) $\sqrt{100 - 36} = \sqrt{64} = 8$

7) $\sqrt{(-\mathbf{3})^2} = \mathbf{3}$

$$\boldsymbol{a^{m/n} = \sqrt[n]{a^m}}$$

8) $16^{1/2} = 4$ * ½ exponent is square root

9) $(-27)^{2/3}$ DROP THE NEGATIVE, then put it in your calculator using the ∧ for exponent and key and $\frac{5}{4}$ as a fraction using $A^{b/c}$ key

10) $\left(\frac{25}{64}\right)^{3/2}$ $\frac{125}{512}$ put in calculator using A b/c key as fractions

11) Watch the video that describes Unifying Root Functions.

Click here to watch the video.

In the video for Unifying Root Functions, how many solutions were there to the equation solved?

Choose the correct answer below.

- A. Two
- B. Three
- C. One ✔
- D. None

12) $\sqrt{2x+3} = 5$ square both sides, drops radical on the left

$2x + 3 = 25 \rightarrow 2x = 22 \rightarrow x = 11$

13) $\sqrt{2x-4} = -6$ *NO SOLUTION BECAUSE SQUARE ROOT ≠ NEGATIVE*

14) $\sqrt[3]{2x-2} + 5 = -1 \rightarrow \sqrt[3]{2x-2} = -6$ cube both sides

$2x - 2 = -216$

$2x = -214$ $x = -107$

15) $\sqrt{x-15} = 6$ square both sides

$x - 15 = 36$

$x = 51$

16) Solve.

$\sqrt{5x} = -2$ *NO SOLUTION BECAUSE SQUARE ROOT ≠ NEGATIVE*

Select the correct choice below and fill in any answer boxes present in your choice.

- A. x = ☐
  (Simplify your answer. Use a comma to separate answers as needed.)
- B. There is no solution. ✔

17) $\sqrt{5x-9} - 2 = 2$ move -2 to the right

$\sqrt{5x-9} = 4$ square both sides

$5x - 9 = 16$ $5x = 25$ $x = 5$

18) $\sqrt[3]{x-7} - 2 = 0 \rightarrow \sqrt[3]{x-7} = 2$ cube both sides

$x - 7 = 8$

$x = 15$

19) $x = 12\sqrt{x}$ *Square both sides* $x^2 = 144x$

$x^2 - 144x = 0$

x(x-144) = 0

x = 0, 144

20) $\sqrt{18 - 3x} = x$ *square both sides* $18 - 3x = x^2$

$0 = x^2 + 3x - 18$

$(x - 3)(x + 6) = 0$

x = 3, ~~-6~~

square root can not be a negative number

x = 3

21) $x = 2\sqrt{2x - 4}$ *square both sides* $x^2 = 4(2x - 4)$

$x^2 = 8x - 16$

$x^2 - 8x + 16 = 0$

$(x - 4)(x - 4) = 0$

x = 4

22) 2+ $\sqrt{4x - 3} = x$ *move 2 to the right and*

$4x - 3 = x - 2$ *square both sides *use FOIL on the right*

$4x - 3 = x^2 - 4x + 4$

0=$x^2 - 8x + 7$

(x-7)(x-1)

x = 1,7

CHECK BOTH ANSWERS

2+ $\sqrt{4(1) - 3} = (1)$

4 ≠ 1 NO

2+ $\sqrt{4(7) - 3} = (7)$

2+5 = 7 yes x = 7

23) $\sqrt{4(x+7)} - 4 = x$ *move -4 to the right and*

$\sqrt{4x+28} = x + 4$ *square both sides *use FOIL on the right*

$4x + 28 = x^2 + 8x + 16$

$0 = x^2 + 4x - 12$

(x+6)(x-2)

x = -6,2

CHECK BOTH ANSWERS $-6 + 4 = \not{-}2$ $2 + 4 = 2$

NO ↖ yes x = 2

square root can not be a negative number

24) $(2x+2)^{1/2} = 8$ *square both sides* $2x + 2 = 64$

$2x = 62$

x = 31

25) $(4x+1)^{1/3} = 5$ *cube both sides* $4x + 1 = 125$

$4x = 124$

x = 31

26) $x^{3/2} - 40x^{1/2} = 0$ *Use u substitution* *$u^3 - 40u = 0$*

$u = x^{1/2}$ *$u(u^2 - 40) = 0$*

$u = 0$ $u^2 = 40$

$X^{1/2} = 0$ $(X^{1/2})^2 = 40$

X = 0, 40

27) $y^{\frac{1}{2}} - 3y^{\frac{1}{4}} + 2 = 0$

Let u = $x^{\frac{1}{4}}$ equation $u^2 - 3u + 2 = 0$

SHORTCUT:

*raise factored answer to the 4^{th} power

(u-2)(u-1) = 0

u = 1 and u = 2

$x^{\frac{1}{4}} = 1$ $\quad$ $x^{\frac{1}{4}} = 2$

x = 1 $\quad$ $x = 2^4 = 16$

28) $x^{\frac{9}{4}} - 2x^{\frac{9}{8}} + 1 = 0$

Let u = $x^{\frac{9}{8}}$ equation $u^2 - 2u + 1 = 0$

(u-1)(u-1) = 0

u = 1

$x^{\frac{9}{8}} = 1$

x = 1

29) $x^{\frac{1}{3}} + 3x^{\frac{1}{6}} - 4 = 0$

Let u = $x^{\frac{1}{6}}$ equation $u^2 + 3u - 4 = 0$

SHORTCUT:

*raise factored answer to the 6^{th} power

(u+4)(u-1) = 0

u = -4 $\quad$ and $\quad$ u = 1

$x^{\frac{1}{6}} = -4$ $\quad$ $x^{\frac{1}{6}} = 1$

$(-4)^6 = 4096$ $\quad$ $(1)^6 = 1$

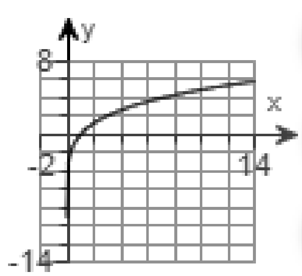
plug both in to check:

$4096^{\frac{1}{3}} + 3 \cdot 4096^{\frac{1}{6}} - 4 = 0$ $\quad$ $1^{\frac{1}{3}} + 3 \cdot 1^{\frac{1}{6}} - 4 = 0$

16 ≠ 0 $\quad$ 0 = 0

X = 1

Behaves like $\sqrt{x}$



EXTRA EXAMPLES

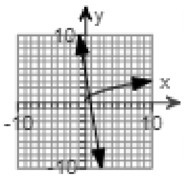
A) Begin by drawing a rough sketch to determine the number of real solutions for the equation $y_1 = y_2$. Then, solve this equation by hand. Give the solution set and any extraneous values that might occur. Do not use a calculator.

$y_1 = \sqrt{x}$

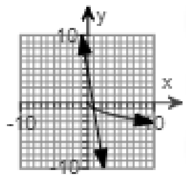
$y_2 = 6x - 5$

Choose the correct graph below.

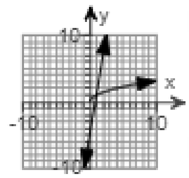
A.



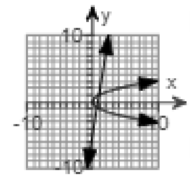
B.



C. ✔



D.



Look at graph where graphs cross

The equation has 1 real solution(s).
(Type a whole number.)

The solution set is $\{1\}$.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

$\sqrt{x} = 6x - 5$ square both sides

$x = 36x^2 - 60x + 25 \qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$0 = 36x^2 - 61x + 25 \qquad x = \dfrac{61 \pm \sqrt{121}}{2(36)} = \dfrac{61 \pm 11}{72} = \dfrac{72}{72} = 1 \qquad \dfrac{61 - 11}{72} = \dfrac{50}{72} = \dfrac{25}{36}$

Are there any extraneous values? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

✔ A. The extraneous values is/are $\dfrac{25}{36}$.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

Short cut: is 6x – 5: square both and put first # on bottom

B. There are no extraneous values.

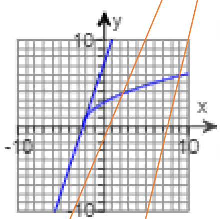
B)

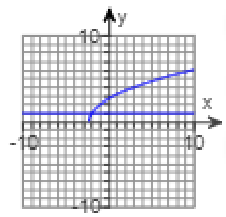
Use an analytic method to solve the equation in part (a). Support the solution with a graph. Then use the graph to solve the inequalities in parts (b) and (c).

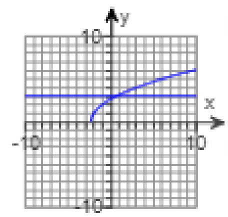
a. $\sqrt{3x+7} = 1$
b. $\sqrt{3x+7} > 1$
c. $\sqrt{3x+7} < 1$

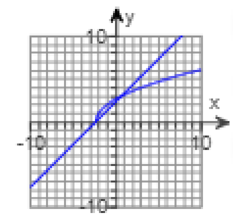
a. The solution set is $\{-2\}$. $3x+7 = 1$ $3x = -6$ $x = -2$
(Simplify your answer. Use a comma to separate answers as needed.)

Support the solution above with a graph. Choose the graph below that represents the solution(s) to the equation $\sqrt{3x+7} = 1$.

A.


B.


C.


D.


b. Use the correct graph above to help solve the inequality $\sqrt{3x+7} > 1$.

$(-2,\infty)$ (Simplify your answer. Type your answer in interval notation.)

c. Use the correct graph above to help solve the inequality $\sqrt{3x+7} < 1$.   pick out graph that matches this

$\left[-\frac{7}{3},-2\right)$ (Simplify your answer. Type your answer in interval notation.)

$3x + 7 = 0$

$x = -7/3$

C)

Use an analytic method to solve the equation in part (a). Support the solution with a graph. Then use the graph to solve the inequalitie parts (b) and (c).

a. $\sqrt{4x+97} = 2x-1$
b. $\sqrt{4x+97} > 2x-1$
c. $\sqrt{4x+97} < 2x-1$

$4x + 97 = (2x-1)^2$

$4x + 97 = 4x^2 - 4x + 1$

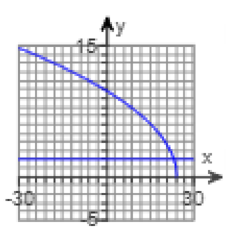
$0 = 4x^2 - 8x - 96$ divide all by 4

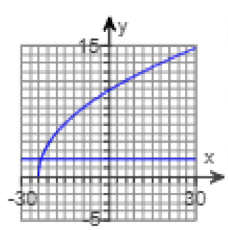
$0 = x^2 - 2x - 24$ $(x-6)(x+4) = 0$

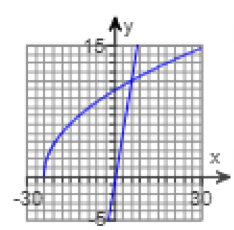
$x = 6, -4$

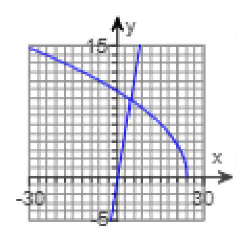
a. The solution set is $\{6\}$.
(Simplify your answer. Use a comma to separate answers as needed.)

Support the solution above with a graph. Choose the graph below that represents the solution(s) to the equation $\sqrt{4x+97} = 2x-1$.

A.


B.


C.


D.


b. Use the correct graph above to help solve the inequality $\sqrt{4x+97} > 2x-1$.

$4x + 97 = 0 \quad x = -97/4$

$\left[-\frac{97}{4}, 6\right)$ (Simplify your answer. Type your answer in interval notation.)

c. Use the correct graph above to help solve the inequality $\sqrt{4x+97} < 2x-1$.

$(6, \infty)$ (Simplify your answer. Type your answer in interval notation.)

D) Use an analytic method to solve the equation in part (a). Support the solution with a graph. Then use the graph to solve the inequalitie parts (b) and (c).

a. $\sqrt{17x+2} + 3 = 2x$
b. $\sqrt{17x+2} + 3 > 2x$
c. $\sqrt{17x+2} + 3 < 2x$

$17x + 2 = (2x-3)^2$

$17x + 2 = 4x^2 - 12x + 9$

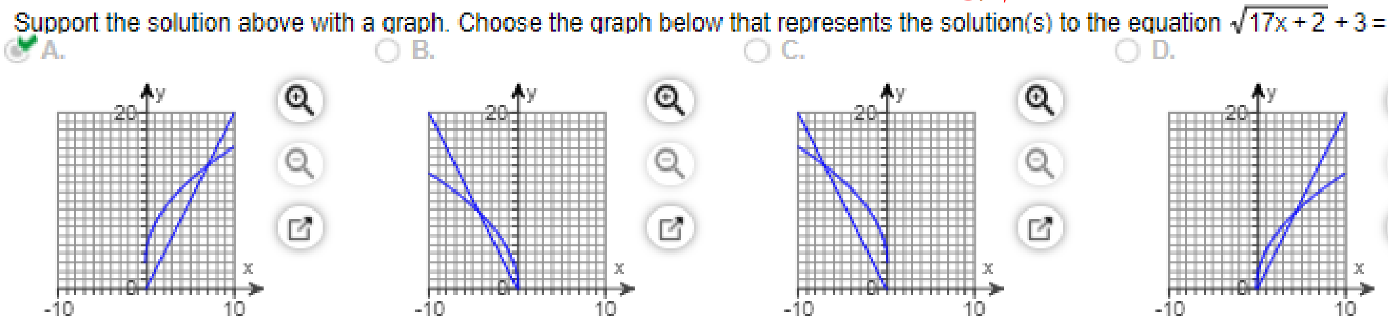
$0 = 4x^2 - 29x + 7$ use slide and divide

$0 = x^2 - 29x - 28$ $(x-28)(x+1) = 0$ divide by 4

$x = 7, \frac{1}{4}$

a. The solution set is $\{7\}$.
(Simplify your answer. Use a comma to separate answers as needed.)

Support the solution above with a graph. Choose the graph below that represents the solution(s) to the equation $\sqrt{17x+2} + 3 = 2x$.



b. Use the correct graph above to help solve the inequality $\sqrt{17x+2} + 3 > 2x$. Solve under radical $17x+2=0$

$x = -\frac{2}{17}$

$\left[-\frac{2}{17}, 7\right)$ (Simplify your answer. Type your answer in interval notation.)

c. Use the correct graph above to help solve the inequality $\sqrt{17x+2} + 3 < 2x$.

$(7, \infty)$ (Simplify your answer. Type your answer in interval notation.)

E) Use analytic or graphical methods to solve the inequality.

$\sqrt{-x} < 0$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

can't have negative under the radical

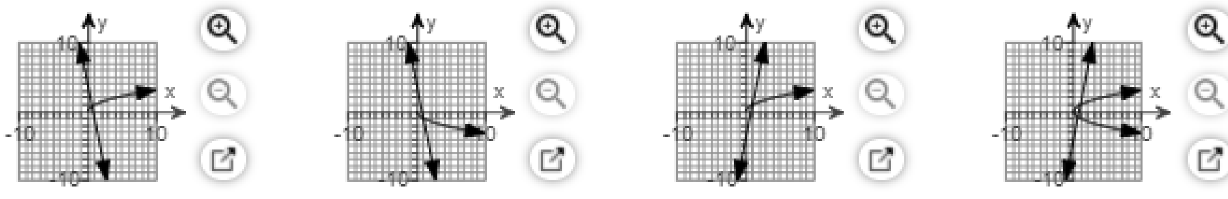
- A. The solution set is ☐.
  (Type your answer in interval notation.)
- ✔ B. The solution set is $\varnothing$.

F) Begin by drawing a rough sketch to determine the number of real solutions for the equation $y_1 = y_2$. Then, solve this equation by hand. Give the solution set and any extraneous values that might occur. Do not use a calculator.

$y_1 = \sqrt{x}$
$y_2 = 5x - 4$

Choose the correct graph below.

A. B. ✔ C. D.



The equation has 1 real solution(s).
(Type a whole number.)

Look at graph where graphs cross

The solution set is {1}.
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

Are there any extraneous values? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

If you substitute both in, only the x = 1 works

- A. The extraneous values is/are $\frac{16}{25}$.
  (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

Since intersection is not on an integer unit, we have to solve algebraically

$\sqrt{x} = 5x-4$ square both sides

$x = 25x^2 - 40x + 16$

$0 = 25x^2 - 41x + 16$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

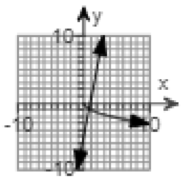
$$x = \frac{41 \pm \sqrt{81}}{2(25)} = \frac{41 \pm 9}{50} : \textit{solve both } \frac{50}{50} = 1 \quad \frac{41-9}{50} = \frac{16}{25}$$

G) Begin by drawing a rough sketch to determine the number of real solutions for the equation $y_1 = y_2$. Then solve this equation by hand. Give the solution set and any extraneous values that might occur. Do not use a calculator.

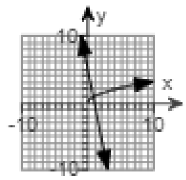
$y_1 = \sqrt{x}$
$y_2 = -5x + 5$

Choose the correct graph below.

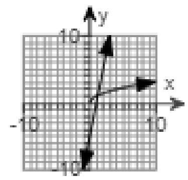
A.



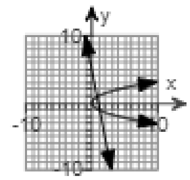
B. ✔



C.



D.



The equation has 1 real solution(s).
(Type a whole number.)

Since intersection is not on an integer unit, we have to solve algebraically

$\sqrt{x} = -5x + 5$ square both sides

$x = 25x^2 - 50x + 25$

$0 = 25x^2 - 51x + 25$

use quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{51 \pm \sqrt{51^2 - 4(25)(25)}}{2(25)} = \frac{51 \pm \sqrt{101}}{50}$$

The solution set is $\left\{ \frac{51 - \sqrt{101}}{50} \right\}$. If you substitute both in, only the - one works

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

Are there any extraneous values? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The extraneous values is/are $\frac{51 + \sqrt{101}}{50}$. Same but switch the sign

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

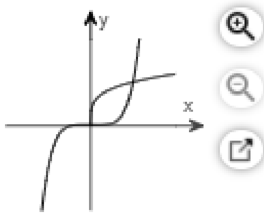
H) Begin by drawing a sketch to determine the number of real solutions for the equation $y_1 = y_2$. Then solve this equation by hand. Give the solution set and any extraneous values that might occur.

$y_1 = \sqrt[4]{x}$

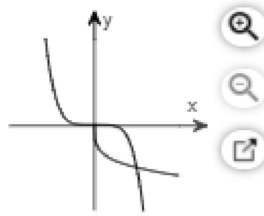
$y_2 = x^5$

Choose the correct sketch of y_1 and y_2.

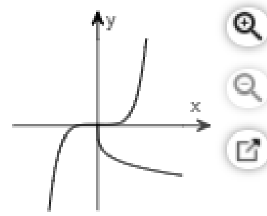
✔ A.



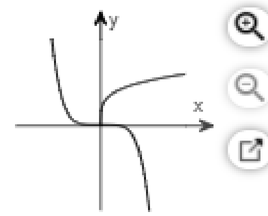
○ B.



○ C.



○ D.



The equation has 2 real solutions.
(Type a whole number.)

Solve this equation by hand and give the solution set.

Since intersection is not on an integer unit, we have to solve algebraically

$\sqrt[4]{x} = x^5$ raise both sides to the 4th power

$x = x^{20}$

$0 = x^{20} - x$ factor out an x

$0 = x(x^{19} - 1)$ X = 0 and 1

If you substitute both in, both work

The solution set is $\{0,1\}$.
(Simplify your answer. Use a comma to separate answers as needed.)

Are there any extraneous values?

○ A. The extraneous values is/are ___.
(Simplify your answer. Use a comma to separate answers as needed.)

✔ B. There are no extraneous values.

I) $x^{\frac{5}{2}} - 2x^{\frac{5}{4}} + 1 = 0$ Let u = $x^{\frac{5}{4}}$ equation $u^2 - 2u + 1 = 0$

$(u-1)(u-1) = 0$

u = 1 then $x^{\frac{5}{4}} = 1$

x = 1

J) $x^{\frac{3}{4}} - 16x^{\frac{3}{8}} + 64 = 0$

Let u = $x^{\frac{3}{8}}$ equation $u^2 - 16u + 64 = 0$

(u-8)(u-8) = 0

u = 8 then $x^{\frac{3}{8}} = 8$ put in $8^{8/3}$

x = 256